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**Short Note**

**Stability Analysis of Linear Reactor Systems with Reactivity Fluctuations**

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As regards theoretical studies of stochastic reactor analysis with parametric multiplicative noises it has been remarked by several authors\(^{(1)}\)\(^{\sim} \)\(^{(4)}\) the appearance of an increase on reactivity. As a consequence, when the reactor model is linear, fluctuations in reactivity reduce the deterministic stability region. This is related with a more general behavior of linear systems submitted to multiplicative noises that consists in the existence of unstable moments\(^{(2)}\). The comprehension of the mechanism of production of these instabilities can be interesting in order to understand the anomalous fluctuations experimentally observed in certain nuclear reactors, which have been associated with the existence of parametric fluctuations\(^{(3)}\).

The first attempt to explain the divergence of the moments was done by Williams (1971) who remarked that instabilities do not appear when the input is a periodic deterministic reactivity. Then he considered two possible causes: the unbounded character of the used noise (Gaussian), or the existence of zero frequencies in the spectral density of the noise. Williams dealt with a bounded noise (clipped Gaussian), which apparently leads also to the divergence of the moments of high order and so he concluded that boundness is not the cause of moments instability.

Nowadays it is argued that the simple Langevin approach can be incomplete to describe correctly a nuclear reactor, and the problem is related to the Ito-Stratonovich ambiguity\(^{(6)}\). It is not our purpose to discuss this delicate problem of fundamentation. We restrict our attention to external noise where Stratonovich interpretation is the only valid\(^{(7)}\).

In the present note we consider again bounded noises and we study in detail the stability of the moments. From our results the mechanism of production of instabilities can be understood. For this purpose we take a simple stochastic reactor model with reactivity fluctuations described by the stochastic differential equation:

\[
\frac{dn}{dt} = -((\lambda - A)n - \xi n + s), \quad (1)
\]

where \(n\) is the neutron density, \(\lambda\) and \(A\) are the capture and fission rates respectively, \(s\) is an independent density source and \(\xi\) a multiplicative noise in the capture rate. This equation can be obtained by doing the thermodynamic limit in a rigorous description of internal and external fluctuations\(^{(8)}\), where \((\lambda + \xi)n\) is the capture transition probability and \(N\) the number of neutrons in the reactor. The positivity of the transition probability implies the positivity of \((\lambda + \xi)\) and so the bounded character of the noise. For computational purposes it is usually assumed that \(\xi\) is a Gaussian noise. With this approach exact results are possible\(^{(9)}\)\(^{(10)}\), but it is clear that it does not modelize, in a rigorous way, a physical system. In the following we shall deal with two kinds of bounded noises which also allow us to obtain exact results. One of them is a white Poisson noise which modelizes a instantaneous pulsed fluctuation and the other is a dichotomic noise which modelizes a two level fluctuation. Both noises can represent many physical phenomena included as sources of reactivity fluctuations.

The Poisson white noise is characterized by the Poisson parameter \(L\) and by the pulse amplitude distribution \(w(S)\) which by simplicity we assume to be exponential:

\[
w(S) = (1/S_0) \exp(-S/S_0), \quad (2)
\]

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where $S_o$ is the amplitude mean value\textsuperscript{(11)}. In this case the Poisson white noise with zero mean is bounded by $-S_o L$, $\xi(t) \geq -S_o L$, and physical conditions implies that $\lambda - S_o L \geq 0$.

The dichotomic noise is characterized by the correlation time $t_c$ and by the amplitude $\pm \Delta$. This noise is bounded by $\pm \Delta$, $-\Delta \leq \xi(t) \leq \Delta$, and physical conditions implies that $\lambda - \Delta \geq 0$. If we use these simple noises for $\xi$ in Eq. (1) it is possible to obtain the stationary probability of the neutron density.

We apply the general expressions given in Refs. (11) and (12) to our Eq. (1) and obtain for the Poisson white noise\textsuperscript{(11)}:

$$p_{\xi}^{\xi}(n) = N_p n^{s/S_o} \left[ s - (\lambda - S_o L - A) n \right]^{s/(\lambda - S_o L - 1)} - 1, \quad (3)$$

where $N_p$ and the support region for $p_{\xi}^{\xi}(n)$ are

$$N_p = \frac{\left( \lambda - S_o L - 1 \right)^{1+1/S_o} B \left( \frac{L}{\lambda - S_o L - 1}, 1 + \frac{1}{S_o} \right)}{(s) \left( \lambda - S_o L - 1 \right)^{s/(\lambda - S_o L - 1) - 1}}, \quad n \in \left( 0, \frac{s}{\lambda - S_o L - 1} \right), \quad (4)$$

when $(\lambda - S_o L - A) > 0$, and

$$N_p = \frac{\left( A + S_o L - 1 \right)^{1+1/S_o} B \left( \frac{L}{A + S_o L - 1}, 1 + \frac{1}{S_o} \right)}{(s) \left( A + S_o L - 1 \right)^{s/(A + S_o L - 1) - 1}}, \quad n \in \left( 0, \infty \right), \quad (5)$$

when $(\lambda - S_o L - A) < 0$.

If we consider the dichotomic noise we have\textsuperscript{(13)}

$$p_{\xi}^{\xi}(n) = N_p n^{(s - (\lambda + A - A) n - s) \left[ \lambda - (\lambda - A) n \right]^{t(\lambda - A - s) - 1}} \left[ s - (\lambda - A - A) n \right]^{t(\lambda - A - s) - 1}, \quad (6)$$

where $N_p$ and the support region for $p_{\xi}^{\xi}(n)$ are given by

$$N_p = \frac{(A - A - \lambda)^{1+1/S_o} B \left( \frac{L}{A - \lambda - 1}, 1 + \frac{1}{S_o} \right)}{(s) \left( A - \lambda - 1 \right)^{s/(A - \lambda - 1) - 1}}, \quad n \in \left( 0, \frac{A - A - \lambda}{\lambda - A - A} \right), \quad (7)$$

when $A < \lambda - A$ and by

$$N_p = \frac{(A - A + \lambda)^{1+1/S_o} B \left( \frac{L}{A + \lambda - 1}, 1 + \frac{1}{S_o} \right)}{(s) \left( A + \lambda - 1 \right)^{s/(A + \lambda - 1) - 1}}, \quad n \in \left( 0, \frac{A - A + \lambda}{\lambda - A - A} \right), \quad (8)$$

when $A > \lambda - A$. In these expressions $B$ is the Beta function\textsuperscript{(14)}.

In both cases we have considered two regions: in the first one $(4, 7) \lambda - A + \xi(t) > 0$ for any time, and in the other region $(5, 8) \lambda - A + \xi(t) < 0$ with finite probability. In the first region the fact that $\lambda - A + \xi(t) > 0$ is satisfied guarantees that the instantaneous reactivity is always negative, that is, the reactor is in a subcritical state for any time. In the second region the reactor can be supercritical with finite probability. The stability of the moments is very different in each region. In the first region when $\lambda - A + \xi(t) > 0$, all moments are stable and take the values

$$\langle n^k \rangle = \frac{s^k}{\lambda - S_o L - A} \left( \frac{s}{\lambda - S_o L - A} \right)^{k/(1/S_o + 1)}, \quad (9)$$

$$\langle n^k \rangle = \frac{s^k}{\lambda - A + A} \left( \frac{\lambda - A}{\lambda - A + A} \right)^{k/(1/(\lambda - A + A))}, \quad (10)$$
where \( (X)_K = X(X+1) \cdots (X+K-1) \) and \( _2F_1 \) is the hypergeometric function\(^{(1)}\). In the other case only a finite number of moments are stable; for the Poisson noise we have

\[
\langle n^K \rangle^p = \frac{1}{2} \left( \frac{S}{\lambda - A + J} \right)^K \left( \frac{1}{2t_c(\lambda - A - J)} + 1 \right)_K \left( \frac{\lambda - A + J}{\lambda - A - J} \right)_K \cdot \, _2F_1\left( -(K+1), \frac{1}{2t_c(\lambda - A + J)}, \frac{\lambda - A + J}{\lambda - A - J} \right),
\]

for \( K < (\lambda - A)/(t_cJ - (\lambda - A)^2) \). Therefore, only in this region the multiplicative noise exhibits its usual instabilities. We can explain the mechanism of instability due to multiplicative noise with this simple example.

When the instantaneous reactivity \( \rho(t) = (A - \lambda + \xi(t))/(A + \lambda) \) is smaller than zero for any time, the reactor is always subcritical, every realization is bounded and all moments are finite. In the opposite case, when \( \rho(t) > 0 \) for some time the reactor is then supercritical, some realization are unbounded, and the moments of high order diverge as in the Gaussian case\(^{(1)}\). We note that our results are not in contradiction with the ones obtained by Williams with a clipped Gaussian noise. Analyzing in detail the behavior of the moments (Sect. 4 of Williams (1971)), it can be extracted the same conclusions that with our bounded noises. The essential point is to relate the bounds for the noise with the physical parameters of the system. As we have shown the instability of the moments is not due to the existence of zero frequencies in the spectral density of the noise.

We conclude with some remarks of practical value. In a description of reactor fluctuations by means of a stochastic equation it appears the necessity of considering bounded noises. This kind of noises induces, in a linear reactor model, different degrees of stability depending on the values of parameters. When parameters are such that \( \lambda - A + \xi(t) > 0 \) for any time the system is always subcritical and all the moments are stable. In other case the reactor will be in a supercritical state during more or less time and the moments of high order diverge. The minimal conditions to assure some stability for the system would be the finiteness of the two first moments. However in this case we can expect spontaneous intensive fluctuations, just when \( \lambda - A + \xi(t) < 0 \), that is, when moments of high order are unstable.

Finally we note that Gaussian noise approximation can be valid only when parameters of the noise are in the second region. Generalization to more realistic models including temperature effects will be studied.

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**References**


