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A Mathematical Journey

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I was happy to be invited by the editors of this book, my dear colleagues and collaborators Giulia Di Nunno and Fred Espen Benth, to write a short Preface. Maybe they regret it now, when they discover that I have decided to use the opportunity to write a short, personal ‘memoir’ of my mathematical life! As my excuse, the publication of this book is a very special occasion for me, and I could not resist the temptation!

At all times my interest in mathematics has been driven by curiosity and interest. At the same time I have been influenced by the inspiring ongoing research activity around me. Thus I find that my mathematical life can be described as a travel, both through time and space and through mathematics itself. Accordingly, thinking back I find that it is natural to divide my mathematical journey into periods, as below.

1. 1945–1971: the early years, culmination with a PhD at UCLA

Encouraged by my parents, I was already as a boy interested in mathematics, science and philosophy. This interest developed further in high school in Flekkefjord, Norway, where my mathematics and physics teacher, Johannes Lohne, was a renowned researcher in the history of science. In class he often told us interesting stories about famous scientists, their lives and discoveries.

Starting my science studies at the University of Oslo (UiO) in 1964 opened a door to an exciting new world for me. I was inspired by lectures, tutorials and seminars by charismatic scientists, including Karl Egil Aubert, Erik Alfsen and Otte Hustad (in mathematics), Otto Øgrim, Helmut Orimestad and Kristoffer Gjøtterud (in physics) and Jon Medbøe (in cosmology).

Gradually my science interests concentrated more and more on mathematics, particularly on topics within mathematical analysis, and in 1970, supervised by Otte Hustad, I completed my cand. real. (Masters) degree at the University of Oslo, with a thesis on rational approximation, which involved a fascinating combination of complex analysis, potential theory and functional analysis. The problems that I, and several of my fellow students, studied at that time were related to uniform approximation of non-analytic functions by means of algebras of analytic functions. Central concepts in this research are peak sets, interpolation sets, capacities and Hausdorff measures. For example, the classical F. & M. Riesz theorem states that a continuous function on a closed subset $K$ on the boundary of the unit circle in the complex plane $\mathbb{C}$ can be approximated uniformly by continuous functions which extend analytically to the interior of the circle (i.e. the unit circle).
disc) if and only if $K$ has Lebesgue measure 0. In one of my first papers [7] I gave a short and elementary proof of this result. Later Sandy Davie (from Dundee University, later Edinburgh University) and I extended this to the unit ball of $C^n$ and obtained what was later termed the Davie-Øksendal theorem ([11], Section 10.4).

My work on the cand. real. thesis in Oslo was inspired by brilliant works by Sandy Davie and also by fundamental works by Ted Gamelin and John Garnett, both from the University of California, Los Angeles (UCLA). In 1970 I got a scientific assistant grant from UiO and this, which was later followed up by a research assistant grant from UCLA, made it possible for me and my fellow UiO student Otto Bekken, to start PhD studies at UCLA in August 1970.

Coming to UCLA was fantastic! Suddenly I was surrounded by excellent mathematicians (including my supervisor Ted Gamelin) and fellow students, in a genuine creative research environment. At the same time the qualifying courses made sure that we got well trained in all mathematical subjects, including real analysis, complex analysis, algebra, topology, geometry, probability and logic. This gave me an enormous inspiration and energy in my research, and I was able to pass all the qualifying exams and defend my PhD thesis at UCLA by June 1971. Moreover, this marvelous academic year was topped by my wife Eva giving birth to our first child in February.

2. 1972–1981: probability and analysis

In the years that followed I first spent one year as a post doc at the University of Edinburgh, then 15 months in military service at Kjevik, Norway before I got a position at the University of Agder in Kristiansand. In this period I started getting interested in probability theory and its interplay with classical mathematical analysis, particularly the part which is related to the potential theory aspects of rational approximation. A fundamental problem in potential theory is the famous Dirichlet problem, which can be stated as follows:

Given a continuous function $f$ on the boundary $\partial D$ of a given bounded open set $D$ in $\mathbb{R}^2$, find a continuous function $\tilde{f}$ on the closure $\bar{D}$ of $D$, such that $\tilde{f} = f$ on $\partial D$ and $\tilde{f}$ is harmonic on $D$.

It is well-known that (a mild interpretation of) this problem always has a unique solution $\tilde{f}$, and the value of $\tilde{f}(x)$ at a point $x \in D$ can be obtained by integrating the boundary function $f$ with respect to a boundary measure $\mu_x$, i.e.

$$\tilde{f}(x) = \int_{\partial D} f(y) d\mu_x(y).$$  \hfill (2.1)

This measure $\mu_x$ is called the harmonic measure for the point $x$. Since it is so fundamental for the Dirichlet problem, it is of interest to study the properties of this measure. A celebrated result of Kakutani states that this harmonic measure $\mu_x$ coincides with the first exit distribution from $D$ of Brownian motion $B(t)$ starting at the point $x$. I was fascinated by this result and applied it to prove that harmonic measure is always singular with respect to area measure [8]. (Note that in general the boundary of a domain in the plane may have positive area.) This result turned out to be true in more general contexts, and it is sometimes called Øksendal’s theorem [5].

In the spring of 1982 I was invited by Alan Sinclair and Sandy Davie to visit the University of Edinburgh to give an introductory course on stochastic differential equations (SDEs). I did not know anything about SDEs beforehand, but decided to take the challenge. It turned out to be a rewarding experience, both because of the valuable feedback from the audience and because the topic was so interesting, with a fascinating interplay between theory and applications, which included optimal stopping and stochastic control. My lecture notes from this course were later polished on several occasions, and in 1983 they were submitted to Catriona Byrne, who was a newly appointed editor at Springer. She believed in the project and decided to publish it in the Springer Universitext series, where the first edition of my book SDEs [9] appeared in 1985. The book became very popular, probably because of its informal style. It filled a gap in the literature, which up to then consisted mostly of (excellent, but) technical presentations of the subject. During this decade I was appointed professor at UiO, and for several years I had the pleasure of giving courses based on my book for a number of talented, bright students. Some of them, I am happy to say, are now my colleague.


In 1991 I was appointed adjunct professor (a 20% position) at the Norwegian School of Economics (NHH) in Bergen, and this gave me an excellent opportunity to study the fascinating interplay between stochastic analysis and finance.

In addition, in the early part of this period I started fruitful cooperation with colleagues at UiO (Helge Holden, Tom Lindstrom, Jan Ubøe and Tusheng Zhang) on white noise calculus and applications to stochastic partial differential equations (SPDEs) and the study of fluid flow in porous media. This was a research project funded by VISTA, a research cooperation between the Norwegian Academy of Science and Letters (DNVA) and the Norwegian state oil company Statoil. In the years 1992–1996 I was appointed VISTA professor. This work culminated in the book [6], the first edition of which appeared in 1996. The same year I was elected a member of DNVA and awarded the Nansen Prize for my research in stochastic analysis and applications.

In 1993 my life got a new and great dimension, when Knut Aase from the Norwegian School of Economics (NHH), Bergen, and I visited the University of Botswana (UB) to attend the Southern African Mathematical Sciences Association (SAMSA) Conference that year. There, together with Mark Roberts and David Henwood at the University of Zimbabwe (UZ) and Edward Lungu (UB), we worked out the plans for a Master and PhD collaboration project between UiO and SAMSA. The project started in 1996, funded by NUFU in Norway. This project was very successful. During its 10 years of running, it produced more than 50 Master students and 10 PhD’s who later entered important positions in the region. Later Giulia Di Nunno joined me as the Norwegian coordinator of the project, which was subsequently taken over by other, similar projects between UiO and countries in the SAMSA region.
5. 2001–2016: further ongoing developments

In this period, for me all the previous theories and methods came together to form a very fruitful interplay. Most of the projects started in this period are still ongoing.

- For example, Agnès Sulem (INRIA, Paris) and I started working on applications of stochastic control to finance. This project has resulted in a number of papers, and also the book [10].
- Francesca Biagini, Yaozhong Hu and I started working on fractional Brownian motion and applications, a collaboration which culminated in the book [3].
- Knut Aase, Nicolas Privault and Jan Ubøe and I had already some years earlier started working on applications of white noise theory to finance, see [1,2]. In the following years, with Giulia Di Nunno and Frank Proske we applied the white noise machinery to develop and study the Hida–Malliavin calculus and its applications [4].

6. Conclusion

In 2014 I was awarded the University of Oslo Research Prize. I appreciate highly this recognition of my work. At the same time I feel that all I have done is to live a rich and happy life in pursuit of my curiosity and interest, being inspired by marvellous environments, collaborators and friends all over the world. I have also been blessed by my wonderful wife Eva, who for more than 50 years has faithfully stood by me and supported me.

References